ECE 313 Lecture Notes

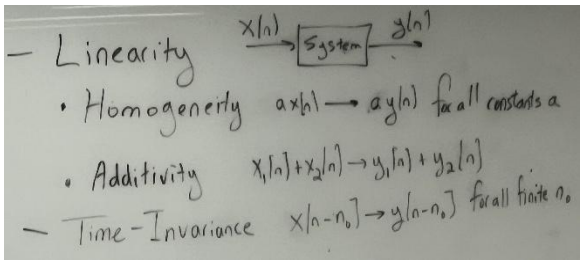
Lecture 9 on Frequency Responses Part 1

Tuesday, October 20, 2026

By TA Alice Liu

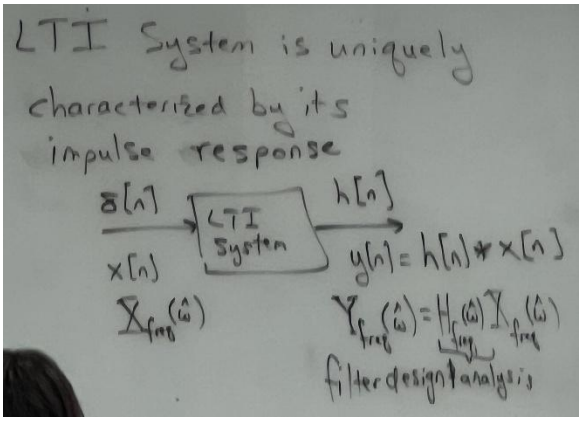
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Review of last week

- An LTI system satisfies…
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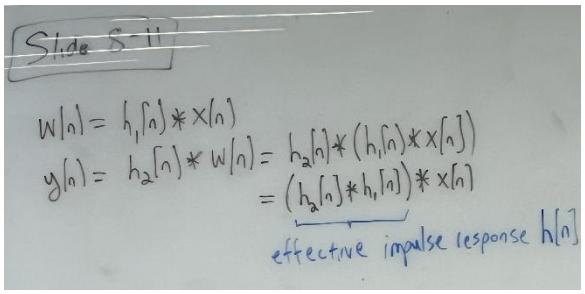
  - Linearity
    - Homogeneous
    - Additivity
  - Time-Invariance
  - Why do these properties matter?
    - Any system can have an impulse response .. only LTI systems are COMPLETELY/ UNIQUELY characterized by their impulse response! Via Discrete-Time convolution
    - Convolution in Time means Multiplication in Frequency
    - Duality: Convolution in Frequency means Multiplication in Time
    - Connection to Probability: When adding two discrete statistically independent random variables x and y, i.e. z = x + y, the probability density function for z is the convolution of the PDFs of x and y:

$$f_Z(z) = f_X(z) * f_Y(z)$$



Discrete Time Convolution

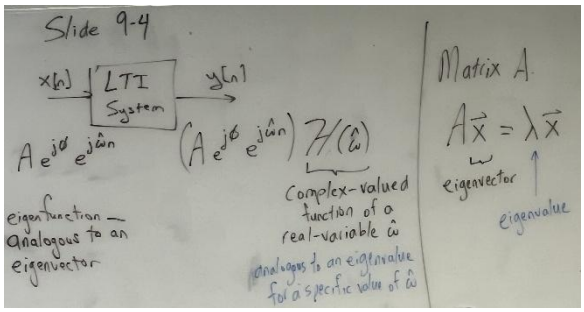
- Convolution Properties
  - Impulse: $x[n] * \delta[n - n_0] = x[n - n_0]$ for constant n_0
  - Commutative: x convolved with h is the same as h convolved with x
    - MATLAB demonstration: 2-point Averaging Filter impulse response convolved with 10-sample rectangular pulse
  - Associative: if you convolve 3 signals together and perform pair-wise convolution, the order in which you convolve the pairs does not matter
    - Cascaded LTI Systems (application of associative property)



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Sinusoidal Response of FIR Filters

- Impulse response == the same as the filter coefficients
- Transfer function (for frequency response)
  - Relates output to input
  - There is also a transfer function for Z-domain (more to come later)
  - Eigen function:
  - Output is the input (eigenvector) scaled by the complex value (eigenvalue)



- Frequency Response
  - Taking advantage of polar form, take advantage of collecting like terms...
    - To obtain magnitude response & phase response (a.k.a. phase-shift) as a response to frequency
  - How many frequencies should we plot?
    - Since the discrete-time frequency domain is periodic with period 2π, we only have to plot frequencies in range of $[-\pi, \pi]$
    - Due to symmetry, we only have to plot frequencies in range $[0, \pi]$ because the magnitude response is even symmetric about zero frequency and phase response is odd symmetric about zero frequency. This is what the Matlab command freqz does.

Tune-Up – Impulse Response

